

# Asymmetry in Complex Numerals

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# Purpose

I discuss the properties of complex numerals in different languages and focus on the computational procedure ( Merge + recursion) deriving them.

I focus on the additive and the multiplicative semantics of complex numerals and raise the question whether it follow from natural language semantics, or whether it also requires the intervention of other aspects of cognition.

Finally, I consider current work on the biological implementation of this procedure, that is, the activation of specific neuronal networks.

# Questions

I ask the question why complex numerals are human specific.

I consider the role of asymmetry in the derivation and the interpretation of complex numerals.

I discuss the properties of numeric expressions with overt operators and address the question of why complex numerals are always asymmetrical.

I discuss the properties of silent operators in complex numerals and raise the question why these silent operators are absent in syntax.

Finally, I raise the question whether the conceptual system interfacing with numeric expressions coincides with the conceptual-intentional system.

# Research agenda

I aim to understand grammar within a broader biolinguistic framework, on the basis of notions that have been shown to shed light on the dynamics of complex systems such as biology and physics, namely the notions of symmetry, asymmetry and symmetry-breaking. These notions, we claim, may provide conceptual unification between language and biology.

- formal properties of the operations of FLN  
(the asymmetry of Merge)
- language variation  
(symmetry>anti-symmetry>asymmetry )
- factor reducing derivational complexity  
(symmetry breaking)

In this talk, I explore the properties of complex numerals within this broader research program, and consider the relation between the properties of the language faculty and its connections with arithmetic from a biolinguistic perspective.

# Biolinguistics

Study of the biology of language

Faculty of Language

## **Formal properties of language**

The computational procedure giving rise to the discrete infinity of language (Minimalist Grammars, Merge+recursion, Interfaces)

## **Origin of the language**

The emergence of the language faculty (ability to compute complex structures)

## **Language in the architecture of the mind/brain**

The relations between the language faculty and the other cognitive faculties (Dynamic Interfaces)

# Complex numerals

Investigating the properties of complex numerals from a biolinguistic perspective may offer new insights on the specificity of human language, on the emergence of complex numerals, as well as on the connection between language and other cognitive faculties including mathematics.

(1) two hundred, two hundred and one, three thousand two hundred three thousand two hundred and one, ...

The ability to develop complex numerals is human-specific. Thinking beyond experience is a by-product of a uniquely human, non-adaptive, cognitive capacity.

Comparative studies of mathematical capabilities in nonhuman animals indicate that many animals can handle numbers up to 6-7 (perhaps directly, or perhaps via subitizing), but they cannot deal with greater numbers.

What are the properties of complex numerals that makes them human specific?

# GG

Several proposals are available for the analysis of Numerals (cardinals, ordinals). Different views on their category, derivation, interpretation and recursive properties are available.

(Ritter 1991, Li, 1996, 1997, 1998, Li and Sybesma 1999, Yeung 2001; Hurford 1975, 1987, 1998, Comrie 2005, Zabbal 2005, Ionin & Matushansky 2006, Stump 2010, Mengden 2010, Chierchia 2011 a.o.)

# Category

The majority of cardinals are Nouns.

(Hurford 1975, 2003)

Numerals are functional heads heading NUMP projections

(Ritter 1991)

Low numerals are adjectives, high numerals are nouns

(Zweig 2004, Zabbal 2005)

The status of numerals varies across languages, nevertheless, numeral phrases are universally NPs

(Zweig 2005)

Cardinals are nouns functioning as nominal predicates.

(Corver & Zwarts 2006)

Cardinals (both, simplex and complex, e.g. *four books*, *four hundred books*, are nouns. They are semantically modifier, viz.,  $\langle\langle e,t \rangle, \langle e,t \rangle\rangle$  categories, and take an NP argument. (Ionin & Matushansky 2006)

Cardinals are adjectives in Modern Greek and they are generated in NUMP.

(Stavrou & Terzi 2008)

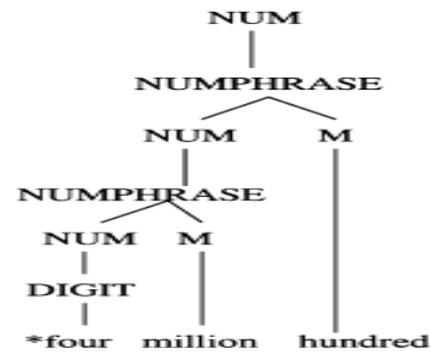
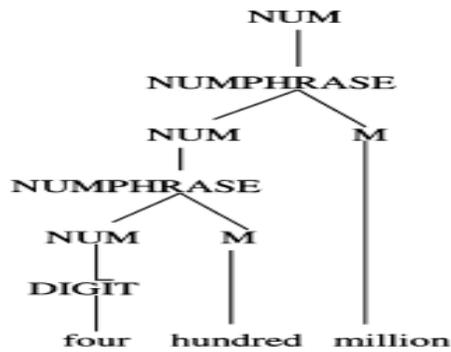
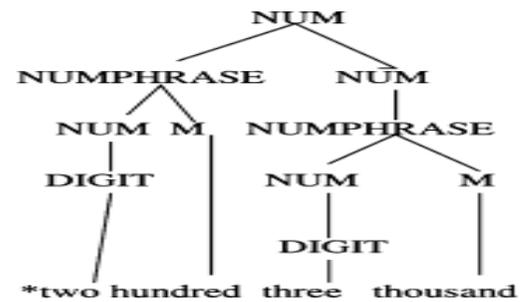
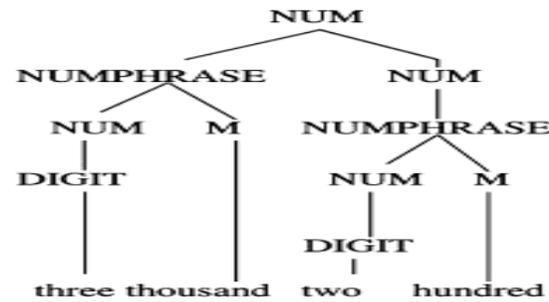
# Hurford 1975

Phrase structure grammar :

(2)  $NUM \rightarrow \{ DIGIT$

$NUMPHRASE (NUM) \}$

$NUMPHRASE \rightarrow NUM M$  Hurford (1975)

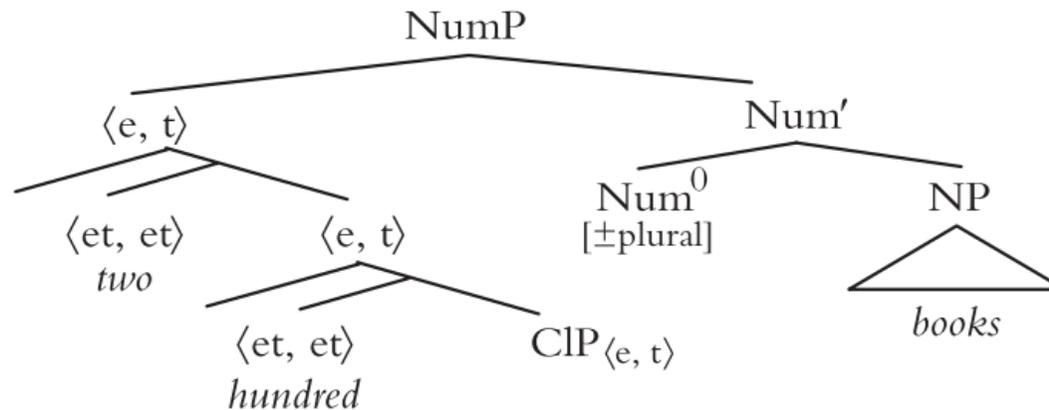




## Ionin & Matushansky 2004, 2006

### X-bar structure and type theoretical semantics

(5)



Complex cardinals involving multiplication (*two hundred*) are analyzed as complementation, whereas complex cardinals involving addition (*one hundred and two*) are two simplex cardinals combined into one via coordination.

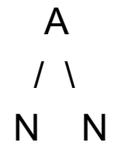
# Zabbal 2005

Argues against Ritter's (1991) functional head hypothesis and Ionin & Matushansky (2004) right node rising analysis.

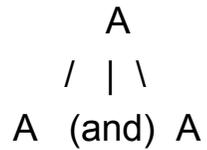
Provides an analysis of Case and agreement in Arabic complex numerals

Proposes a flat structure for bipartite numerals (Zabbal 2005, Zweig 2004, Hurford 2003),

Multiplicative structures (two hundred)



Additive structures (twenty one)



Proposes the introduction of a new semantic type N for numerals compositional semantics.

## Two biolinguistic questions

I raise the following fundamental questions:

- What are the properties of the generative procedure deriving complex numerals?
- How is this procedure biologically implemented?

These fundamental questions were either covert or absent from the early studies in generative grammar. The biolinguistic program brings them to the fore.

Investigating the syntactic-semantic properties of numerals from a biolinguistic perspective may offer new insights on the specificity of human language as well as on the connection between language and mathematics.

# Hypotheses

Complex numerals are derived by the computational procedure of the language faculty.

Their processing activate the same neuronal network than the processing of syntactic expressions (adjunction and coordination)

The core aspect of their semantics comes out from natural language semantics (compositionality).

Certain aspects of their interpretation comes from other systems than Cl.

1. Recursion in complex numerals and argue that it is mediated by a functional projection.
2. They are composed by Asymmetric Merge
3. Parts of their interpretation activate the neuronal network differently.

# 1. Recursive procedure

1.1 bounded/unbounded recursion

1.2 (in)direct recursion

# Human specific

The ability to develop complex numerals, and thus recursive complex numerals, is human-specific.

It has been argued that numerals do not present the same recursive properties than language, and that English number names is finite. We discuss the properties of one language which provide empirical evidence of unbounded infinity of number-names, which were hypothesized for very large numbers in English.

If number-names are unbounded, that is if they are infinite, they share a basic property of language.

# (Un)bounded recursion

It has been argued that numerals do not present the same recursive properties than language. Merrifield (1968) and Greenberg (1978) take the view that there is an upper limit on linguistically expressible number-names.

Greenberg (1978, p. 253) states the following generalization: "Every language has a numeral system of finite scope." Brainerd (1971) take a different view.

"The collection of numerical expressions in most languages, as in English, are basically finite. Thus in English we must ultimately coin new '*illions*' if we are to transcend our [finite] system of number names. And where are these to come from when we have run out of Latin prototypes? For example in Chinese, *wan* is used for  $10^4$  and *wan wan* for  $10^8$ . Presumably we can continue ad infinitum, *wan wan wan*  $10^{12}$ , *wan wan wan wan*  $10^{16}$ , etc. There is evidence that the generative procedure deriving them exhibit unbounded recursion." Brainerd (1971, p. 208)

There is at least one human language, Chinese, where number names show unbounded recursion.

# Chinese numbers

The examples in (6a) and (6b) are well-formed Chinese number-names, while (c) is not. Similarly, (6d) is well-formed, while (6e) is not. (Radzinsky 1991)

- (6) a. wu zhao zhao wu zhao  
five trillion trillion five trillion (5,000,000,000,000,005,000,000,000)
- b. wu zhao zhao zhao zhao zhao wu zhao zhao  
five trillion trillion trillion trillion trillion five trillion trillion  
zhao zhao wu zhao zhao zhao wu zhao zhao wu zhao  
trillion trillion five trillion trillion trillion five trillion trillion five trillion
- c. \*wu zhao zhao wu zhao zhao zhao  
five trillion trillion five trillion trillion trillion
- d. wu zhao zhao zhao zhao wu zhao zhao  
five trillion trillion trillion trillion five trillion trillion
- e. \*wu zhao zhao wu zhao zhao wu zhao zhao zhao zhao  
five trillion trillion five trillion trillion five trillion trillion trillion trillion

The well-formed number-names follow a pattern in which larger clusters of *zhao* precede, from left to right, smaller clusters of *zhao*, (7), while the ill-formed number-names do not adhere to such a requirement.

$$(7) J = \{wu\ zhao^{k_1}\ wu\ zhao^{k_2}\dots, wu\ zhao^{k_n} \mid k_1 > k_2 > \dots > k_n > 0\}$$

The ordering of the clusters might be attributed to interface properties.

# English numbers

Zwicky (1963) discusses some constructions of names for cardinal numbers that are not generated by a CFG. The one he labels (8) resembles the structure of very large number-names in English (and other natural languages):

(8)  $NT^n(, NT^{n-1})\dots (, NT)(, N)$

In this construction, N indicates a number between 1 and 999, T is an abbreviation for *thousand*, commas indicate an intonational pause, and everything within parentheses is optional. This construction could be characterized as follows:

(i) Given a system in English, for example, where *thousand* is used as the largest single word for a number, *million* would be represented as *thousand thousand*, (Amer.) *billion* as *thousand thousand thousand*, (Amer.) *trillion* as *thousand thousand thousand thousand*, etc., *ad infinitum*.

(ii) In a system like (i), larger clusters of *thousand* must precede smaller clusters of *thousand* in the same manner that *decillion* must precede *trillion*, which must precede *million*, which must precede *thousand* in the standard English number-name system using single-words for numbers of higher values.

# Recursion in numerals and concatenation

Numerals cannot be derived by operations on strings such as concatenation,\* since concatenation does not keep track of the properties of the concatenated elements. Furthermore it does not derive hierarchical constituent structure.

\*Concatenation is a function that forms a single string of symbols from two given strings by placing the second after the first.

\*The concatenation operation on strings is generalized to an operation on sets of strings as follows: For two sets of strings  $S_1$  and  $S_2$ , the *concatenation*  $S_1S_2$  consists of all strings of the form  $vw$  where  $v$  is a string from  $S_1$  and  $w$  is a string from  $S_2$ .

(9) twenty one

[one hundred] and [twenty one]

[one thousand] [one hundred and twenty one]

[one million] [one thousand] [one hundred and twenty one]

[one billion] [one million] [one thousand] [one hundred and twenty one]

...

# Indirect recursion in additive structures

Hierarchical constituent structure is signaled by intonational pauses.

Indirect recursion is evidenced by intervening functional projections.

In Romance languages as well as in other languages, including English, a coordinating conjunction must be projected in some cases, (10a) whereas it may be projected in other cases, (10c).

- (11) a. vingt et un (Fr)  
          'twenty and one'  
      b. \*vingt un  
          'twenty one'  
      c. cent (et) un  
          'hundred (and) one'

- (10) vingt et un (Fr)  
      deux cent vingt et un  
      deux mille deux cent vingt et un  
      deux millions deux mille deux cent vingt et un  
      deux milliards deux millions deux mille deux cent vingt et un

Numerals combine via an (c)overt functional category in recursive additive structures

# Indirect recursion in multiplicative structures

Multiplicative structures are also asymmetrical even though there is no overt functional project between the conjuncts. However, empirical evidence that multiplicative structure also includes a functional projection comes from complex multiplicative structures in Romanian, where the preposition DE (of), which is used independently in pseudo-partitive structures, must be part of the recursive multiplicative structures (12a).

- (12) a.   doua sute           de mii                   de carti (Ro)  
          two hundred-PL DE thousand-PL DE books  
          'two hundred thousands books'
- b.   doua sute        de carti  
          two hundred   DE books  
          'two hundred books'
- c.   o mie            de carti  
          one thousand DE books  
          'one thousand books'

## de vs. din/dintre

In Romanian, the preposition *de* appears only with pseudo-partitives, while the preposition *din/dintre* appears only with true partitives.  
(Brasoveanu 2007)

- (12) d. zece grame **de** briză (de capră)  
ten grams of cheese (of goat)  
'ten grams of goat cheese'
- e. zece grame **din** această briză (de capră)  
ten grams of this cheese (of goat)  
'ten grams of this goat cheese'
- f. #zece grame **din** briză (de capră)
- g. #zece grame **de** această briză (de capră)

## Genitive 'de'

The preposition DE is also used for the genitive constructions denoting types instead of tokens, as the following example illustrate

- (12) h. cartile de studenti  
books, the DE student  
'the student books = the books for students'
- i. manual de clasa 1  
manual of first grade  
'first grade manual'

Neither DE nor DIN are possible for genitive construction denoting tokens. The genitive case is spelled out as affix on the possessor however.

- (12) j. cartile studentilor  
books, the students, Gen  
'the books of the students'
- k. the student's books  
cartile studentului  
'books, the student, Gen'

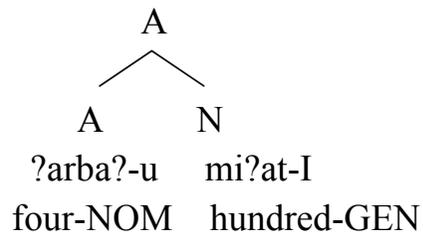
# Case in Arabic complex numerals

Structural/ inherent case in Arabic complex numerals (Zabbal 2005)

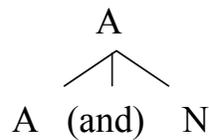
- (13) a. arba –**u**      mi at –**in**      rajul-**in** (Ar)  
four-NOM    hundred-GEN    men-GEN  
'400 men'
- b. arba –**u**      aalaaf- –**in**      rajul-**in**  
four-NOM    thousand-GEN    men-GEN  
'4000 men'
- c. arba –**u**      mi at –**in**      alf-in      rajul-**in**  
four-NOM    hundred-GEN    thousand-GEN    men-GEN  
'400,000,000 men'
- (14) a. arba –**at-u**      aalaaf- –**in**      wa- xams-u      mi at-in    rajul-in    rajul-**in**  
four-NOM    thousands-GEN    and five-NOM    hundred-GEN    man-GEN  
'4500 men'
- b. arba –**at-u**      aalaaf- –**in**      wa- xams-u      mi at-in    rajul-in    wa sitt-at-u    rijaal -**in**  
four-NOM    thousands-GEN    and five-NOM    hundred-GEN    and six-FS-NOM    men-  
GEN  
'4506 men'

# Additive, multiplicative recursive structures

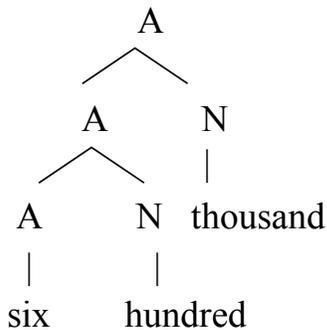
There are empirical and theoretical reasons not to analyze additive and multiplicative structures without intervening functional heads, as flat structures, and recursive multiplicative structures as cases of direct recursion, from Zabbal (2005).



multiplicative structure



additive structure



recursive multiplicative structure

# Summary of section 1

Unbounded recursion is a property of the language faculty.

Unbounded recursion is observed in complex numerals

The recursion is indirect

coordination conjunction, prepositions and Case

The recursive procedure deriving complex numerals is the recursive procedure of the language faculty.

Indirect recursion is human specific.

## 2. Composition

2.1. Additive structures

2.2 Multiplicative structures

# Coordinating conjunctions

Complex numerals in Romance languages may include a coordinating conjunction.

- (16) venti e um (Port)  
vingt et un (Fr)  
douăzeci și unu (Ro)  
twenty and seven  
'twenty one'
- (17) treinta y siete (Sp)  
thirty and seven  
'thirty seven'
- (18) cento e uno (It)  
hundred and one  
'one hundred and one'

# Variation

The pronunciation of the coordination conjunction is subject to variation. In some languages (e.g. Russian), numerical expressions never contain an overt conjunction, in others (e.g. Arabic), an overt conjunction is obligatory for addition (Zabbal 2005). Yet, in other language (e.g. English, French, Italian) the conjunction can, must or may be pronounced. However, the conjunction for multiplication is silent.

- (19) a. vingt et un (Fr)  
b. ventuno (It)  
c. twenty one
- (20) a. deux cent un (Fr)  
b. due cento uno (It)  
c. two hundred and one
- (21) a. trois mille deux cent vingt et un (Fr)  
b. tre mila due cento ventuno (It)  
c. three thousand two hundred twenty one

# Asymmetric coordination

Number names conjuncts are asymmetrical. In a given language, their parts cannot be inverted without giving rise to gibberish (21), (22) or a difference in interpretation, (23), in which case the derivations are distinct.

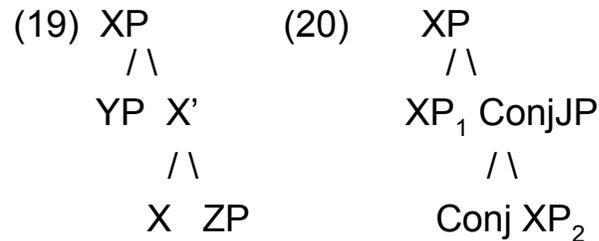
- (21) a. vingt et un/ \*un et vingt (Fr)  
b. ventuno /\*unoventi (It)  
c. twenty one / \*one twenty

- (22) a. einundzwanzig /\* zwanzigundein (Ge)  
‘one-and-twenty’ ‘twenty-and-one’  
b. zweiundzwanzig /\*zwanzigundzwei  
‘two-and-twenty’ ‘twenty-and-two’

- (23) a. deux cent vs. cent deux (Fr)  
b. due cento vs. cento due (It)  
c. two hundred vs. one hundred and two

# Coordination

Coordinations are assumed to be asymmetric structures (Kayne 1994), under an X-bar analysis (Kayne, 1994; Munn, 1987; Johannessen, 1998) or under an adjunction analysis (Munn 1993). According to the adjunction approach, XP is a projection of the first conjunct XP and XP dominates XP. The structure of coordination is asymmetric.



The presence of conjunctions in number names indicate that they are asymmetric hierarchical structures. Given Minimalist assumptions, feature valuation applies in the derivation of conjunctions as it does in the derivation of syntactic structures more generally.

(Kayne 1994, Cinque 1999, Rizzi 1999, Munn 1987, 2003, Johannessen 1998, Chierchia, Partee, Turner 1989, Chierchia & Mc Connell-Ginet 2000, Velde 2006, Agafonova 2010, a.o.)

# Asymmetric Merge

External Merge and Internal Merge are also uniquely human to the extent that it also derives the unobservable, namely the feature asymmetry between the merged elements. This property of Merge have not been found in the form on non-human expressions.

(13) *Merge* (Chomsky 1995)

Target two syntactic objects  $\alpha$  and  $\beta$ , form a new object  $\Gamma \{\alpha, \beta\}$ , the label LB of  $\Gamma$  ( $LB(\Gamma) = LB(\alpha) \text{ or } LB(\beta)$ ).

(14) *Asymmetry Morphology* (Di Sciullo 2005)

Morphological Merger combines trees.

Feature checking applies under asymmetric Agree.

(15) *The Asymmetry of Merge* (Di Sciullo and Isac 2008)

Merge is an operation that applies to a pair of elements in the Numeration whose sets of features are in a proper inclusion relation.

*a. Asymmetry of External Merge* External Merge is an operation that applies to a pair of elements in the Numeration whose categorial features are in a proper inclusion relation.

*b. Asymmetry of Internal Merge* Internal Merge is an operation that applies to a pair of elements in the workspace whose (total set of) features are in a proper inclusion relation.

# Conjunctions, a Minimalist account

(1) Conj : [Conj], [uX<sub>1</sub>], [uX<sub>2</sub>]

(2) Numeration { XP<sub>1</sub>, Conj, XP<sub>2</sub> }

(3)

$$\begin{array}{c}
 \text{XP}_2 \\
 / \ \backslash \\
 \text{XP}_2 \quad \text{ConjP} \\
 [X_2] \quad / \ \backslash \\
 \text{Conj} \quad \text{XP}_1 \\
 [\text{Conj}] \quad [X_1] \\
 [uX_1] \\
 [uX_2]
 \end{array}$$

(4) a. Merge (Conj, XP<sub>1</sub>) : { Conj, XP<sub>1</sub> }

[Conj] [ X<sub>1</sub>]

[ uX<sub>1</sub>]

b. Merge (ConjP, XP<sub>2</sub>) : { ConjP, XP<sub>2</sub> }

[Conj] [ X<sub>2</sub>]

[ uX<sub>2</sub>]

# Asymmetric Merge derivation of complex numerals

The successive steps in building of complex numerals as as *twenty two* and *two hundred* structure are all in compliance with Asymmetric Merge, the derivational steps, given the Numeration below:

$N = \{ \text{twenty} [\text{Num}] \text{ F} [\text{ADD}, u\text{Num}, u\text{Num}] , \text{two} [\text{Num}] \}$

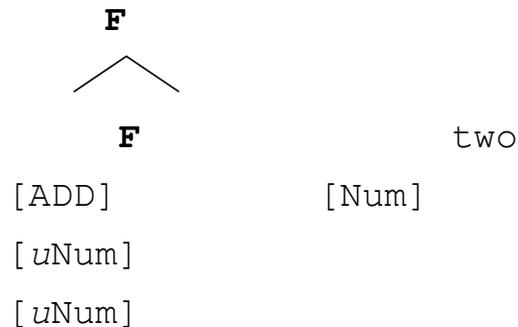
**Step 1.** Select an item from Numeration that has interpretable features only

$\Rightarrow$  Select *two* {[Num]}

**Step 2.** Select an item from Numeration that properly includes *two*

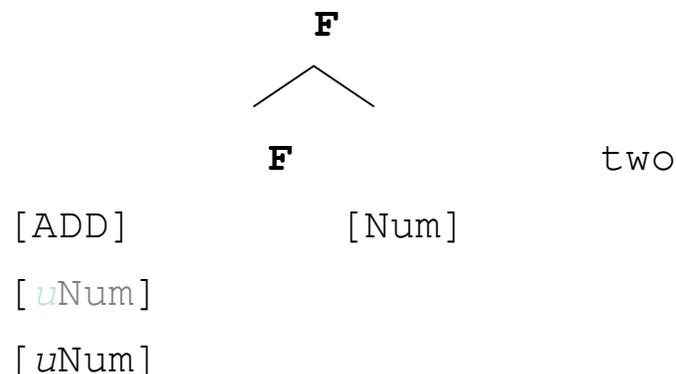
$\Rightarrow$  Select F { [ADD, uNum, uNum ] }

**Step 3.** External-Merge *two* with F.



# Derivation

Given the Earliness Principle (Pesetsky & Torrego 2001), the uninterpretable feature of Num will get checked and erased as soon as possible.



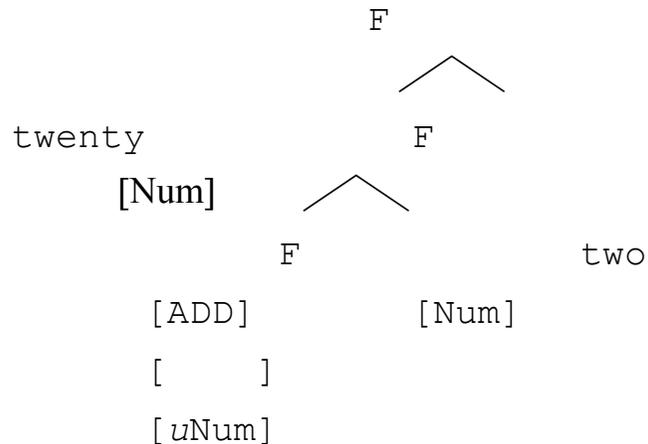
Checking and deletion of uninterpretable features is subsequent to actual Merge. Even though after checking and deleting uninterpretable features, the proper inclusion relation may not hold, what is important is that it holds at the point where Merge applies, i.e. before checking and deletion of uninterpretable features of the newly merged item.

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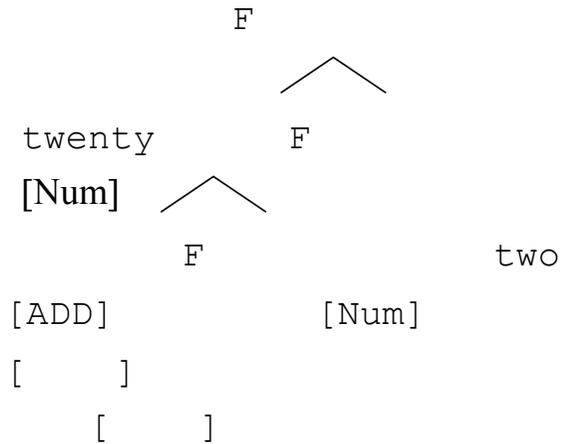
Step 4. Select an item from Numeration that has interpretable features only

⇒ Select *twenty* {[Num]}



# Derivation

**Step 5.** External-Merge *twenty* to the workspace and check uninterpretable features, as enforced by the Earliness Principle.

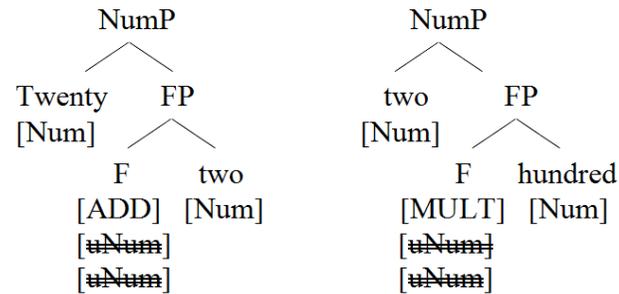


The uninterpretable features are eliminated, and the interpretable features are legible by the external systems. I will assume that the denotation of a numeral is a natural number. The interpretation of a complex numeral is derived compositionally from the interpretation of its parts. Thus, the interpretation of a complex numeral is also a natural number.

# Uninterpretable features

Numerals (NUM) merge with functional projections with interpretable features (ADD, MULT) and uninterpretable features (uNUM). Uninterpretable features are checked and eliminated; interpretable features are legible by the external systems.

(29)



# Interpretable features

The derivation of multiplicative structures is parallel to the derivation of additive structures. The F head is associated to interpretable features restricted to addition (ADD) and multiplication (MULT). The derivations differ with respect to the feature composition of the items undergoing Asymmetric Merge, in particular the position of the interpretable [Base] feature.

(27) a. [ twenty [ F two ] ]  
          ADD

      b. [ two [ F hundred ] ]  
          MULT

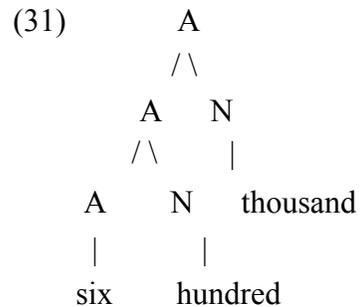
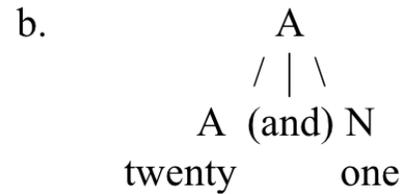
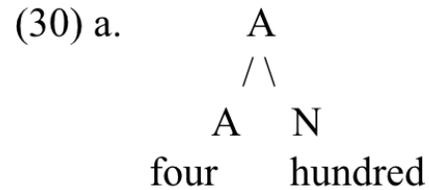
(28) a. [ [ two [ F hundred ] ] [ (and) [ twenty [ F two ] ] ] ]  
          MULT                  ADD                  MULT

      b. [ [ two [ F thousand ] ] (and) [ two [ F hundred ] ] [ (and) twenty [ F two ] ] ]  
          MULT                  ADD          MULT                  ADD          ADD

Complex numerals may include unpronounced heads ADD and MULT legible at the conceptual interface. ADD and MULT features enable the conceptual interpretation of numerals.

# No flat structure and direct recursion

There are theoretical and empirical arguments against a flat structure and direct recursion for complex numerals, (30), (31).



# Indirect recursion

(32) a.

NUMP1  
/\nNUMP FP  
/\nADD NUMP  
twenty two

b.

NUMP2  
/\nNUMP FP  
/\nMULT NUMP  
two hundred

c.

NUMP  
/\nNUMP2 FP  
/\nADD NUMP1  
two hundred and twenty one

d.

NUMP  
/\nFP NUMP  
/\nNUMP2 F  
MULT  
two hundred thousand

## Summary of section 2

Complex numerals are derived by Asymmetric Merge. The features of the elements undergoing this operations are in a minimal proper sub-set relation.

(Un)pronounced functional heads merged in the derivation of complex numerals, which then presents a particular case of indirect recursion.

Additive and multiplicative structures differ in interpretable features: ADD and MULT, and the position of the BASE feature

Indirect recursion introduces configurational asymmetry in a set of otherwise unstructured numeric terms.

# 3. Interface properties

- 3.1 Configurational asymmetry and the syntax-semantics interface
- 3.2 Residual cases and the interfacing with systems external with the language faculty in the broad sense.

# Configurational asymmetry and natural language semantics

While the syntax derivation of coordinate structures relies on the elimination of unvalued features in proper sub-sets, their semantics relies on set formation.

While syntactic processing reduces the sets of features, semantics forms sets with the remaining interpretable features.

The proposed analysis has consequences for our knowledge of the syntax-semantic interface.

# ADD and MULT

Complex numerals denote natural numbers.

Their semantics is compositional

(Ionin & Matushansky 2006, Zabbal 2005, Chierchia 1998, 2011, a.o)

- What is the interpretation of ADD asymmetrically relating the parts of complex numerals?
- How is MULT interpreted?

# ADD/AND

- ADD conjunction is interpreted as the sum of its parts; this is not necessarily the case for AND.

(36) vingt-et-un ,  $20 + 1 = 21$ , and not the set {20, 1}

(37) Paul et Marie, the set {Paul, Marie}

(38) one plus two, one and two

- (39) a. Barbara and Mat went to the market.  
b. Barbara went to the market and Mat went to the market.  
c. Barbara and Mats wrote an article together.  
d. wrote-an-article-together(Barbara @ Mats)

Numeral ADD do not coincide with phrasal AND, notwithstanding the fact that their functional heads can be pronounced by *and/e/et*.

# Non Boolean 'and'

While Boolean and non Boolean conjunction are observed in syntactic expressions, the coordinating conjunction in complex numerals (ADD) is a non-Boolean conjunction, in the sense of Krifka (1983). The conjunction of two numerals is interpreted as the sum of their parts.

To capture its semantics, Masscy (1976), Link (1983), and Hoeksenl(1983). propose an operation which maps entities onto a new entity, their 'sum' or 'collection'.

" $\oplus$ " is the joint operation

" $\oplus$ " is idempotent ( $a \oplus a = a$ ), symmetric ( $a \oplus b = b \oplus a$ ), and associative  
( $a \oplus [ b \oplus c ] = [ a \oplus b ] \oplus c$ )

## +/-, ADD/ MULT, ADD/AND

- Addition and multiplication are symmetrical, (40), ADD and MULT are asymmetrical, (31). There are symmetrical and asymmetrical AND conjunctions.

(40) 1+2, 2+1

10X20, 20X10

(41) two hundred, hundred (and) two

deux cent, cent deux (Fr)

'two hundred', 'one hundred (and) two'

- Phrasal conjunctions with an unpronounced head cannot be interpreted as the product of their parts; whereas this is the case for complex numerals with an unpronounced MULT head.

(42) a. les nombres deux, cent et mille (Fr.)

'the numbers two, one hundred, and one thousand'

(2, 100, 1,000)

b. deux cent mille (Fr.)

'two hundred thousand'

(2 x 100) x 1,000).

# Differences

- While Boolean and non Boolean conjunction are observed in syntactic expressions, the coordinating conjunction in complex numerals (ADD) is a quasi non-Boolean conjunction.
- Some phrasal AND conjunctions are symmetrical, numeral ADD conjunctions are asymmetrical only.
  
- There are phrasal AND conjunctions cannot be interpreted as the sum of their parts, numeral ADD conjunctions must be.
- Phrasal conjunctions with an unpronounced head cannot be interpreted as the product of their parts; whereas this is the case for complex numerals with an unpronounced MULT head.

# Unpronounced heads

A striking fact about numerals is that while the *addition* operation ADD is in some case pronounced by the coordinating conjunction *and*, the *multiplication* operator MULT never is in the languages under consideration, (43).

- (43) a. twenty two
- b. two hundred
- c. two hundred (and) twenty three
- two thousand two hundred (and) twenty three

This contrasts sharply with the interpretation of covert elements in phrasal syntax, which cannot be interpreted as addition or multiplication.

The name of these operations must be pronounced in phrasal syntax, (44), whereas this is not the case in numeric expressions, which name the results of these operations.

- (44) a. twenty plus two
- b. two times one hundred
- c. vingt plus deux (Fr)
- d. deux fois cent

# Complex numerals and arithmetic

It might be the case that the interpretation of ADD and MULT in complex numerals may access the subsystems of cognition that process mathematical operators, including concepts for numbers as well as arithmetic operations deriving complex numbers on the basis of simpler ones.

(33) dix-sept, dix-huit, dix-neuf ( $10+9$ )

vingt, tr-ente, qua-ante, cinqu-ante, soix-ante ( $6 \times 10$ )

quar-ante-sept ( $(4 \times 10) + 7$ )

soix-ante-dix ( $(6 \times 10) + 10$ ), quatre-vingt ( $4 \times 20$ ), quatre-vingt- dix ( $(4 \times 20) + 10$ )

quatre cent cinquante trois ( $(4 \times 100) + ((5 \times 10) + 3)$ )

soixante et un ( $(6 \times 10) + 1$ ), soixante et onze, cinquante et un

quatre-vingt-un, quatre-vingt-onze ( $(4 \times 20) + 11$ )

Given the restrictions on the semantics of ADD and MULT,  
it might be the case that their interpretation is legible by the part of the brain  
that sub-serves mathematic.

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# Brain imaging studies

Friederici, Bahlmann, Friedrich & Makuuchi's (2011) brain imaging results indicate that processing hierarchically structured mathematical formulae and processing complex syntactic hierarchies in language activates different areas of the brain.

Language is a faculty specific to humans. It is characterized by hierarchical, recursive structures. The processing of hierarchically complex sentences is known to recruit Broca's area.

Comparisons across brain imaging studies investigating similar hierarchical structures in different domains revealed that complex hierarchical structures that mimic those of natural languages mainly activate Broca's area, that is, left Brodmann area (BA) 44/45, whereas hierarchically structured mathematical formulae, moreover, strongly recruit more anteriorly located region BA 47.

The present results call for a model of the prefrontal cortex assuming two systems of processing complex hierarchy: one system determined by cognitive control for which the posterior-to-anterior gradient applies active in the case of processing hierarchically structured mathematical formulae, and one system which is confined to the posterior parts of the prefrontal cortex processing complex syntactic hierarchies in language efficiently.

# Two pathways

MERGE from a cortical network / working memory perspective.

There would be a dichotomy to reason either geometrically or equivalently algebraically in mathematics.

This would imply that MERGE is subserved crucially by two pathways, the one for syntax would pass via the ventrolateral prefrontal cortex, strongly relying on Broca's area, and the other implementation for general reasoning which could integrate multi-sensory information via a dorsal fronto-parietal network, with a strong involvement of the posteriorparietal cortex and the angular gyrus.

# Processing complex numerals

We predict that the processing complex numerals mainly activate Broca's area. The fact that the derivation of numerals shares properties with the derivation of syntactic objects, hierarchical structures, while they interface with different cognitive subsystems than syntactic objects would bring support to the hypothesis that the processing of arithmetic operators is biologically based in the language faculty. Their interpretation however would be determined by cognitive control.

There is evidence that complex numerals and syntactic expressions are derived by Merge and the recursive procedure of the language faculty, while their interpretation activates the neuronal network differently.

The fact that the derivation of numerals shares properties with the derivation of syntactic objects while they interface with different cognitive subsystems than syntactic objects brings support to the hypothesis that arithmetic operations are biologically grounded in the language faculty, while their interpretation access different sub-systems of cognition.

# Summary

We raised the following questions:

What is the computational procedure that derives complex numerals?

How are complex numerals interpreted by the external systems?

We argued that the procedure that derives numerals is Asymmetric Merge and indirect recursion via unpronounced operators.

The hypothesis that mathematics emerged with Merge offers a rationale to the fact that Merge and recursion are observed in arithmetic as well as in language.

However, it might be the case that numerals and phrases are interpreted by different sub-system of the cognition.

The fact that MULT is never pronounced indicates that these operators are accessed directly by the part of the cognitive system that sub-serves mathematics.

Then, numerals would find their biological basis in the neuronal faculty that sub-serves grammar but goes beyond it.

## Concluding remarks

Our hypothesis fits in well with comparative studies of mathematical capabilities in nonhuman animals: many animals can handle with numbers up to 6-7,, but they cannot deal with greater numbers, and they do not have the language faculty.

This kind of exploration is possible in a framework that takes the language faculty to be a generative procedure deriving the discrete infinity of language. It supports the view that complex numerals emerged with Merge and the generative procedure of language faculty.

It offers a rationale to the fact that Merge and recursion are observed in arithmetic as well as in language, while arithmetic and syntactic expressions are interpreted differently.

Complex numerals find their biological basis in the neuronal faculty that sub-serves grammar but go beyond it.

# Summary

I discussed the properties of complex numerals in different languages and focus on the computational procedure deriving them..

## 1. Merge + recursion

Asymmetric Merge and indirect recursion are characteristics of this procedure. Asymmetric Merge is an operator that applies to pairs of elements, the features of which are in a set inclusion relation.

## 2. Recursive procedure

Recursion in complex numerals is indirect, that is, mediated by a functional element, in both additive and multiplicative structures, notwithstanding the fact that there is no direct evidence in certain cases e.g. *one hundred and one, one thousand one hundred and twenty one ...*

## 2. Merger

Complex numerals include a functional projection F with interpretable features, e.g. [ADD] and [MULT] and uninterpretable features, e.g. [uNUM]. Semantic interface differences between additive and multiplicative numerals fall out from differences in configurational asymmetry and natural language semantics, while other differences may come from the properties of other systems external to the language faculty.

## 4. Brain-imaging studies

Brain-imaging results indicate that the brain processes mathematical and syntactic expressions differently. These and further studies may bring support to my claim that complex numerals and syntactic expressions are derived by the computational procedure of the language faculty, while part of their interpretation would activate the neuronal network differently.

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