Biolinguistics, Minimalist Grammars, and the Emergence of Complex Numerals

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Biolinguistics

Study of the biology of language

Formal properties of language

Perspectives on the computational procedure giving rise to the discrete infinity of language

Language evolution

Perspectives on the emergence/evolution of the language faculty

Properties of language development

Language in the architecture of the mind/brain

Perspectives on the relations between the language faculty and the other cognitive faculties

Language as a natural object

Approaches to the relations between language and physics

….

Minimalist Grammars

Minimalist Grammars and the language faculty

- Merge, a binary operator
- Recursion is a property of the computational procedure
- The external systems, semantic and sensory motor, accessed through interfaces

Minimalist grammars and the emergence of the language faculty
Minimalist grammars and human/animal studies
Minimalist grammars and the specificity of the language faculty

The emergence of complex numerals

Comparative studies of mathematical capabilities in nonhuman animals indicate that many animals can handle numbers up to 6-7 (perhaps directly, or perhaps via subitizing), but they cannot deal with greater numbers.

The ability to develop complex numerals is human-specific. Thinking beyond experience is a by-product of a uniquely human, non-adaptive, cognitive capacity.

(1) two hundred, two hundred and one, three thousand two hundred three thousand two hundred and one, …

Investigating the properties of complex numerals from a biolinguistic perspective may offer new insights on the specificity of human language, on the emergence of complex numerals, as well as on the connection between language and other cognitive faculties including mathematics, …

Research agenda

We aims to understand grammar within a broader biolinguistic framework, on the basis of notions that have been shown to shed light on the dynamics of complex systems such as biology and physics, namely the notions of symmetry, asymmetry and symmetry-breaking. These notions, we claim, may provide conceptual unification between language and biology.

-formal properties of the operations of FLN
(the asymmetry of Merge)
-language variation
(symmetry>anti-symmetry>asymmetry)
-factor reducing derivational complexity
(symmetry breaking)

We explore the properties of complex numerals and consider the relation between the language faculty and arithmetics from a biolinguistic perspective.

Numerals in GG

Several proposals are available for the analysis of Numerals (cardinals, ordinals). Different views on their category, derivation, interpretation and recursive properties are available.

Category

The majority of cardinals are Nouns.
(Hurford 1975, 2003)

High numerals are adjectives, low numerals are nouns
(Zabbal 2005)

The status of numerals varies across languages, nevertheless, numeral phrases are universally NPs
(Zweig 2005)

Cardinals are nouns functioning as nominal predicates.
(Corver & Zwarts 2006)

Cardinals (both, simplex and complex, e.g. *four books, four hundred books*, are nouns. They are semantically modifier, viz., <<e,t>, <e,t>> categories, and take an NP argument. (Ionin & Matushansky 2006)

Cardinals are adjectives in Modern Greek and they are generated in NUMP.
(Stavrou & Terzi 2008)

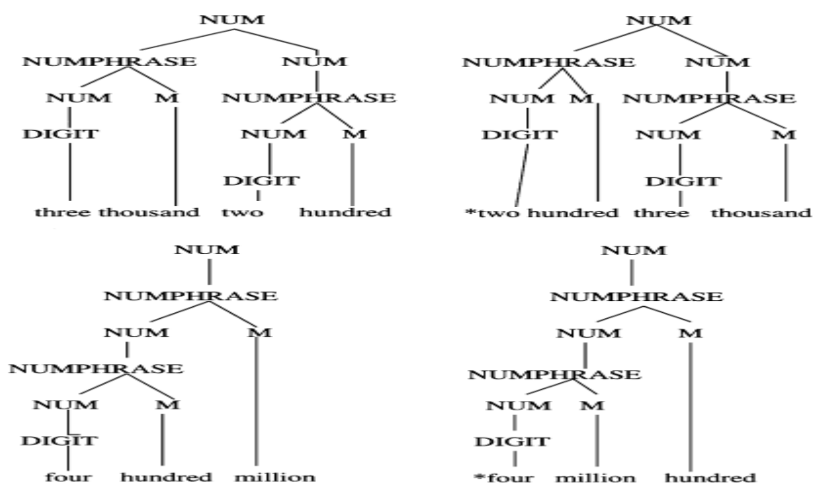
Hurford 1975

Phrase structure grammar :

(2) NUM —> { DIGIT

NUMPHRASE (NUM) }

NUMPHRASE —> NUM M Hurford (1975)



Ionin &Matushansky 2006

X-bar structure and type theoretical semantics (Ionin &Matushansky 2006)

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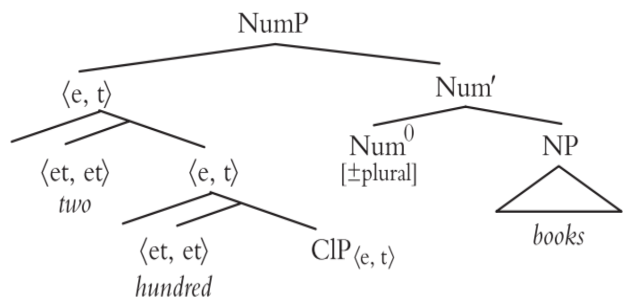

Complex cardinals involving multiplication (*two hundred*) are analyzed as complementation, whereas complex cardinals involving addition (*one hundred and two*) are two simplex cardinals combined into one via coordination.

Stavrou & Terzi 2008

In Greek, Simplex and complex cardinals are generate
in the specifier of NUMP. This accounts for their recursive structure, contra Qs.

With Cardinals containing Numerical Nouns, recursion is located in the complement position of NUMCL. (Stavrou & Terzi 2008),

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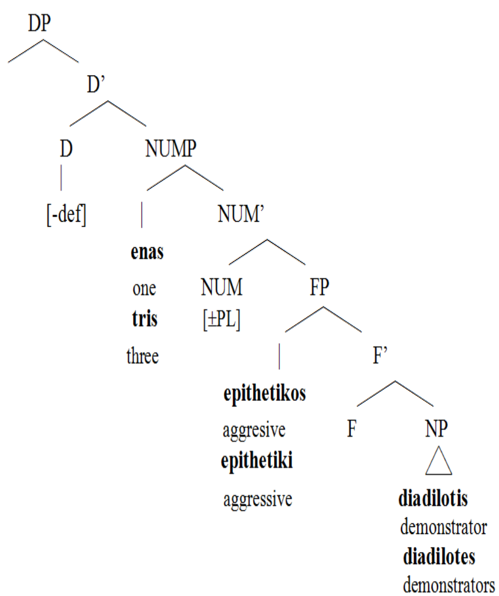

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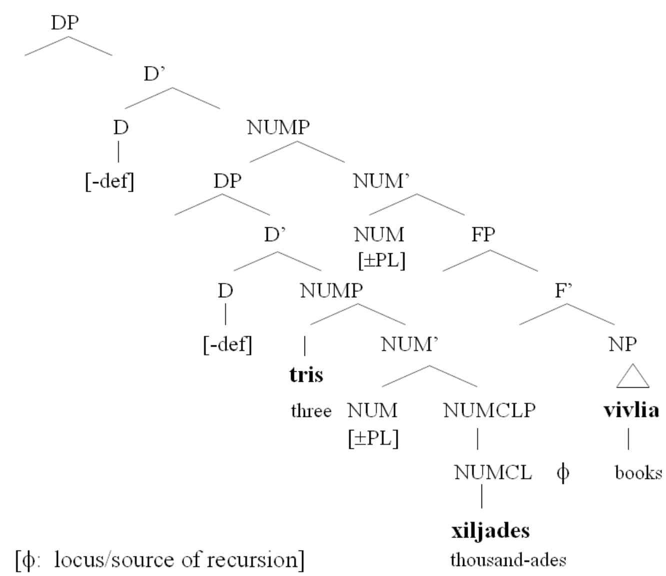


Biolinguistic Questions

We raise the following questions:

- What is the generative procedure deriving complex numerals?
- How is this procedure biologically implemented?

Investigating the syntactic-semantic properties of numerals from a biolinguistic perspective may offer new insights on the specificity of human language as well as on the connection between language and mathematics.

Outline

1. We discuss recursion in complex numerals and argue that it is mediated by a functional projection.

2. We argue that complex numerals are derived by Asymmetric Merge.

3. We argue that complex numerals are grounded in the language faculty but go beyond it.

1. Recursive procedure

The ability to develop complex numerals, and thus recursive complex numerals, is human-specific.

It has been argued that numerals do not present the same recursive properties than language, and that English number names is finite. We discuss the properties of one language which provide empirical evidence of unbounded infinity of number-names, which were hypothesized for very large numbers in English.

If number-names are unbounded, that is if they are infinite, they share a basic property of language.

1. Recursion

Zwicky (1963) discusses some constructions of names for cardinal numbers that are not generated by a CFG. The one he labels (6) resembles the structure of very large number-names in English (and other natural languages):

(6) NT^n(, NT^{n-1})... (, NT)(, N)

In this construction, N indicates a number between I and 999, T is an abbreviation for *thousand,* commas indicate an intonational pause, and everything within parentheses is optional. This construction could be characterized as follows:

(i) Given a system in English, for example, where *thousand* is used as the largest single word for a number, *million* would be represented as *thousand thousand,* (Amer.) *billion* as *thousand thousand thousand,* (Amer.) *trillion* as *thousand thousand thousand thousand, etc., ad infinitum.*

(ii) In a system like (i), larger clusters of *thousand* must precede smaller clusters of *thousand* in the same manner that *decillion* must precede *trillion,* which must precede *million,* which must precede *thousand* in the standard English number-

name system using single-words for numbers of higher values.

(Un)bounded recursion

It has been argued that numerals do not present the same recursive properties than language. Merrifield (1968) and Greenberg (1978) take the view that there is an upper limit on linguistically expressible number-names.

Greenberg (1978, p. 253) states the following generalization: ”Every language has a numeral system of finite scope.” Brainerd (1971) take a different view.

‘’The collection of numerical expressions in most languages, as in English, are basically finite. Thus in English we must ultimately coin new *'illions'* if we are to transcend our [finite] system of number names. And where are these to come from when we have run out of Latin prototypes? For example in Chinese, *wan* is used for 10^4 and *wan wan* for 10^8 . Presumably we can continue ad infinitum, *wan wan wan* 10^{12}, *wan wan wan wan* 10^{16} , etc. There is evidence that the generative procedure deriving them exhibit unbounded recursion.” Brainerd (1971, p. 208)

There is al least one human language, Chinese, where number names show unbounded recursion. The properties in (i) and (ii) described by Zwicky (1963) are attested in human language.

Chinese numbers

The examples in (a) and (b) are well-formed Chinese number-names, while (c) is not.
Similarly, (d) is well-formed, while (e) is not. (Radzinsky 1991)

(7) a. wu zhao zhao wu zhao
five trillion trillion five trillion (5,000,000,000,000,005,000,000,000)

b. wu zhao zhao zhao zhao zhao wu zhao zhao
five trillion trillion trillion trillion trillion five trillion trillion
zhao zhao wu zhao zhao zhao wu zhao zhao wu zhao
trillion trillion five trillion trillion trillion five trillion trillion five trillion

c. *wu zhao zhao wu zhao zhao zhao
five trillion trillion five trillion trillion trillion

d. wu zhao zhao zhao zhao wu zhao zhao
five trillion trillion trillion trillion five trillion trillion

e. *wu zhao zhao wu zhao zhao wu zhao zhao zhao zhao
five trillion trillion five trillion trillion five trillion trillion trillion trillion

The well-formed number-names follow a pattern in which larger clusters of *zhao* precede, from left to right, smaller clusters of *zhao*, (8), while the ill-formed number-names do not adhere to such a requirement.

(8) $J = \{wu\ zhao^{kl}\ wu\ zhao^{k2}..,\ wu\ zhao^{kn} \mid k1 > k2 > ... > kn > 0\}$

Radzinsky shows that neither TAG nor MCTAG can derive Chinese numbers. He also suggest that the ordering of the clusters might be attributed to interface properties.

Recursion in numerals

Numerals cannot be derived by operations on strings such as the concatenation operations, since concatenation does not keep track of the properties of the concatenated elements. Furthermore it does not derive hierarchical constituent structure.

Concatenation is a function that forms a single string of symbols from two given strings by placing the second after the first.

(9) twenty one
one hundred and twenty one
one thousand one hundred and twenty one
one million one thousand one hundred and twenty one
one billion one million on thousand one hundred and twenty one
…

(10) vingt et un (Fr)
deux cent vingt et un
deux mille deux cent vingt et un
deux millions deux mille deux cent vingt et un
deux milliards deux millions deux mille deux cent vingt et un
…

Indirect recursion in additive structures

Hierarchical constituent structure, signaled by the presence of functional categories and intonational pauses indicate that recursion in complex numerals is indirect.

Considering functional categories, a coordinating conjunction must be projected in some cases, whereas it may be projected in other cases.

(11) a. cent vingt et un (Fr)
*cent vingt un
vingt et un (Fr)
'twenty and one'
b. *vingt un
'twenty one'

These facts indicate that numerals (additive structures) may combine via a functional category in additive structures.

If a functional category is projected between two numeral elements, the recursion in complex numeral is indirect. This kind of recursion part of the recursive procedure of the language faculty.

Indirect recursion in multiplicative structures

Multiplicative structures are also asymmetrical even though there is no overt functional project between the conjuncts. However, empirical evidence that multiplicative structure also includes a functional projection comes from complex multiplicative structures in Romanian, were the preposition DE (of), which is used independently in peudo-partitive structures, must be part of the recursive multiplicative structures ().

(12) a. doua sute de mii de carti (Ro)
two hundred-PL DE thousand-PL DE books
'two hundred thousands books'

b. doua sute de carti
two hundred DE books
'two hundred books'

c. o mie de carti
one thousand DE books
'one thousand books'

Case and agreement are observed in complex numerals in other languages as well, including Russian and Arabic.

These facts support the idea that recursion in numerals is indirect. That is, it is mediated by a functional head.

Case and Agreement

Structural/ inherent case and agreement in Arabic complex numerals (Zabbal 2005)

(13) a. arba –**u** mi at –**in** rajul-**in** (Ar)
four-NOM hundred-GEN men-GEN
'400 men'

b. arba –**u** aalaaf- –**in** rajul-**in**
four-NOM thousand-GEN men-GEN
'4000 men'

c. arba –**u** mi at –**in** alf-in rajul-**in**
four-NOM hundred-GEN thousand-GEN men-GEN
'400,000,000 men'

(14) a. arba –at-**u** aalaaf- –**in** wa- xams-u mi at-in rajul-in rajul-**in**
four-NOM thousands-GEN and five-NOM hundred-GEN man-GEN
'4500 men'

b. arba –at-**u** aalaaf- –**in** wa- xams-u mi at-in rajul-in wa sitt-at-u rijaal -**in**
four-NOM thousands-GEN and five-NOM hundred-GEN and six-FS-NOM men-GEN
'4506 men'

Summary of section 1

Unbounded recursion is a property of the language faculty.

There is at least one language exhibiting overt unbounded recursion in numerals.

Indirect recursion is part of the recursive procedure of the language faculty .

Complex numerals exhibit indirect recursion.

- Coordination conjunction
- Case and agreement

The recursive procedure operative in the derivation of complex numerals is the recursive procedure of the language faculty.

2. Computational procedure

The ability to develop complex numerals is human-specific. Thinking beyond experience is a by-product of a uniquely human, non-adaptive, cognitive capacity.

Chomsky's (1988) discussion of counting as an abstraction of the Faculty of Language offers two reasons for which it should be the case,

i) the development of the mathematical ability in different people, and
ii) the improbability of a system exhibiting discrete infinity.

Merge is crucial for counting as well as for thinking about the unobservable.

In what sense is Merge necessary for counting?

Merge

Suppose that a language has the simplest possible lexicon: just one LI, call it ‘one’. Application of Merge to {one} call it ‘two’. Application of Merge to {one} yields {one, {one} } call it ‘three’ and so on. Merge applied in this fashion yields the successor function. (Chomsky 2008)

A question that could be asked is whether this Merge is External or internal. If every application of Merge requires access to the lexicon, then Merge is external.

If we limit the numeration to one item, a single term numeration would permit only one selection. The computational system would require that the system recycle what is already found in the current state of the structural description. That would be Internal Merge. (Bolender 2011)

Several works seems to indicate that External Merge is not human specific (Byne and Russon (1998) mountain Gorilla food preparation, McGonigle et al. (2003) capuchin monkeys ordering objects, Seyfarth et al. (2005) Baboons knowledge of their companions, Schino et al. (2006) Japanese macaques knowledge of their companions, a.o.)

Counting would then be compatible with the hypothesis that Internal Merge, yielding quantification, Operator-variable structures, Opx (...x…), would be human-specific.
Quantificational, Operator-variable structures, are necessary in theoretical languages, including terms that designate unobservable, contra observational languages.

Working memory

Uriagereka (2008) argues that, being a context-sensitive operation, Internal Merge places greater demand on working memory than does External Merge alone.

The probe-goal search, required by Internal Merge, involves scanning the derivational record to find the object to be merged.

External Merge does not require a scan of the derivational history.

So less developed working memory in the other primitive species could suffice to explain their using Merge, but not Internal Merge (Coolidge & Wynn 2005).

Accounting for the human uniqueness of Internal Merge could also explain the human uniqueness of open ended counting, assuming the restricted numeration hypothesis.

Asymmetric Merge

External Merge and Internal Merge are also uniquely human to the extent that it also derives the unobservable, namely the feature asymmetry between the merged elements. This property of Merge have not been found in the form on non-human expressions to my knowledge.

(13) *Merge* (Chomsky 1995)
Target two syntactic objects α and β, form a new object Γ $\{\alpha,\beta\}$,
the label LB of Γ (LB(Γ)) = LB(α) or LB(β).

(14) Asymmetry Morphology (Di Sciullo 2005)
Morphological Merger combines trees.
Feature checking applies under asymmetric Agree.

(15) *The Asymmetry of Merge* (Di Sciullo and Isac 2008)
Merge is an operation that applies to a pair of elements in the Numeration whose sets of features are in a proper inclusion relation.

a.Asymmetry of External Merge External Merge is an operation that applies to a pair of elements in the Numeration whose categorial features are in a proper inclusion relation.

c.Asymmetry of Internal Merge Internal Merge is an operation that applies to a pair of elements in the workspace whose (total set of) features are in a proper inclusion relation.

Given minimalist assumptions, language creativity is the conjunction of the composition operation and a recursive procedure. The hypothesis that composition and recursion are asymmetrical in the sense that composition relies on proper the sub-set relation and that recursion is indirect, can be considered to be part of the uniquely human Theoretical Language

Hypothesis

Complex numerals are derived by Asymmetric Merge.

(Un)pronounced functional heads merged in the derivation of complex numerals, which then presents a particular case of indirect recursion.

Indirect recursion introduces configurational asymmetry in a set of otherwise unstructured numeric terms.

Coordinating conjunctions

Complex numerals in Romance languages may include a conjunction.

(16) venti e um (Port)
vingt et un (Fr)
douăzeci şi unu (Ro)
twenty and seven
'twenty one'

(17) treinta y siete (Sp)
thirty and seven
'thirty seven'

(18) cento e uno (It)
hundred and one
'one hundred and one'

Variation

The pronunciation of the coordination conjunction is subject to variation. In some languages (e.g. Russian), numerical expressions never contain an overt conjunction, in others (e.g. Arabic), an overt conjunction is obligatory for addition (Zabbal 2005). Yet, in other language (e.g. English, French, Italian) the conjunction can, must or may be pronounced. However, the conjunction for multiplication is silent.

(19) a. vingt et un (Fr)
  b. ventuno (It)
  c. twenty one

(20) a. deux cent un (Fr)
  b. due cento uno (It)
  c. two hundred and one

(21) a. trois mille deux cent vingt et un (Fr)
  b. tre mila due cento ventuno (It)
  c. three thousand two hundred twenty one

Coordination

Coordinations are assumed to be asymmetric structures (Kayne 1994), under an X-bar analysis (Kayne, 1994; Munn, 1987; Johannessen, 1998) or under an adjunction analysis (Munn 1993). According to the adjunction approach, XP is a projection of the first conjunct XP and XP dominates XP. The structure of coordination is asymmetric.

```
(19)  XP              (20)     XP
       / \                      / \
    YP  X'                  XP_1 ConjJP
         / \                        / \
       X   ZP                   Conj XP_2
```

The presence of conjunctions in number names indicate that they are asymmetric hierarchical structures. Given Minimalist assumptions, feature valuation applies in the derivation of conjunctions as it does in the derivation of syntactic structures more generally.

Asymmetric conjunctions

Number names conjuncts are asymmetrical. In a given language, their parts cannot be inverted without giving rise to gibberish (21), (22) or a difference in interpretation, (23), in which case the derivations are distinct.

(21) a. vingt et un/ *un et vingt (Fr)
b. ventuno /*unoventi (It)
c. twenty one / *one twenty

(22) a. einundzwanzig /* zwanzigundein (Ge)
'one-and-twenty' 'twenty-and-one'
b. zweiundzwanzig /*zwanzigundzwei
'two-and-twenty' 'twenty-and-two'

(23) a. deux cent vs. cent deux (Fr)
b. due cento vs. cento due (It)
c. two hundred vs. one hundred and two

Conjunctions, a Minimalist account

(1) Conj : [Conj], [uX_1], [uX_2]

(2) Numeration { XP_1, Conj, XP_2 }

(3) XP_2
/ \
XP_2 ConjP
[X_2] / \
Conj XP_1
[Conj] [X_1]
[~~uX_1~~]
[~~uX_2~~]

(4) a. Merge (Conj, XP_1) : { Conj, XP_1}
[Conj] [X_1]
[uX_1]

b. Merge (ConjP, XP_2) : {ConjP, XP_2 }
[Conj] [X_2]
[uX_2]

(Un)interpretable features

The F head is associated with two uninterpreable numeral feaures (uNUM) that need to be check and deleted in the derivation by interpretable features (Num).

The F head is associated to interpretable features restricted to addition (ADD) and multiplication (MULT)

(27) a. [twenty [F two

b. [two [F hundred

(28) a. [two F hundred [(and) [twenty [F three

MULT ADD

b. [two [F thousand [(and) two [F hundred [(and) twenty [F three

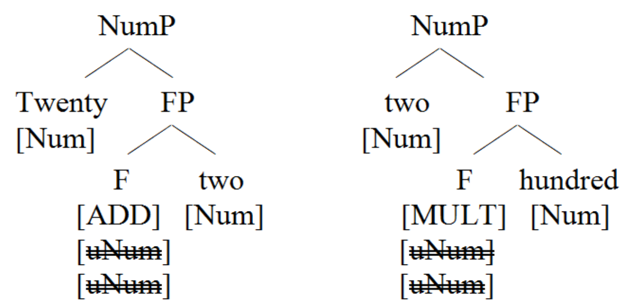
MULT MULT ADD

Complex numerals may include unpronounced heads ADD and MULT legible at the conceptual interface. ADD and MULT features enable the conceptual interpretation of numerals.

Complex numerals

Numerals (NUM) merge with functional projections with interpretable features (ADD, MULT) and uninterpretable features (uNUM). Uninterpretable features are checked and eliminated; interpretable features are legible by the external systems.

(29)



Flat structure and direct recursion

There are theoretical and empirical arguments against a flat structure and direct recursion for complex numerals, (30), (31).

```
(30) a.      A                  b.            A
            / \                             / | \
           A   N                           A (and) N
        four   hundred               twenty        one

(31)          A
             / \
           A     N
          / \    |
        A          thousand
        |      |
       six   hundred
```

Coordinating conjunctions are asymmetrical. (Kayne 1994, a.o.)

Coordinate structure coordinate constituents of the same type.

Adjectives are in the spec of functional projections. (Cinque 1999, a.o.)

A coordinating conjunction is pronounced in additive structures.

A preposition is pronounced in recursive multiplicative structures.

Genitive case is legible is Romanian, Russian and Arabic complex numerals.

Hierarchical structure and indirect recursion

```
(32)             NumP
                  / \
                FP    thousand
                / \
         NumP    F
             / \
          six   FP
                 / \
               F    hundred
```

Summary of section 2

What is the computational/recursive procedure deriving complex numerals?

Asymmetric merge combine numerals indirectly though a functional head.

This functional head has both interpretable and uninterpretable features. Uninterpretable features must be valued in the derivation.

In what sense is Merge necessary for counting and expressing complex numerals?

Merge is necessary for counting and expressing complex numerals as counting and the derivation of complex numerals are based of the operations of the language faculty.

3. Complex numerals and arithmetic

How is the computational procedure deriving complex numerals biologically implemented?

Does the generative procedure that derives complex numerals coincide with the generative procedure that derives mathematic formulae?

How are complex numerals interpreted?

The (un)pronounced functional heads are part of the derivation of complex numerals and contribute to their recursive computation by the operations of the Faculty of Language, as well as their interpretation by the external systems.

Numerals are generated by Merge. They interface with the sub-systems of cognition (Mind/Brain) processing mathematical expressions.

Complex numerals and arithmetic operations

Complex numerals may directly access the subsystems of cognition that process mathematical concepts, including concepts for numbers as well as arithmetic operations deriving complex numbers on the basis of simpler ones.

(33) dix-sept, dix-huit, dix-neuf (10+9)

vingt, tr-ente, qua-ante, cinqu-ante, soix-ante (6X10)

quar-ante-sept ((4 X 10) +7)

soix-ante-dix ((6 X 10)+10), quatre-vingt (4 X 20), quatre-vingt- dix ((4 X 20) +10)

quatre cent cinquante trois ((4 X 100) + ((5 X 10) +3))

soixante et un ((6 X 10) +1), soixante et onze, cinquante et un

quatre-vingt-un,
quatre-vingt-onze ((4 X 20) +11)

Language creativity in numerals is a function of asymmetric merge composing numerals indirectly though unpronounced ADD and MULT operators. ADD and MULT are legible by the part of the cognitive system that sub-serves mathematic.

Some Differences between +/-, ADD/ MULT, ADD/AND

- Addition (+) and multiplication (X) may apply to terms of the same kind; ADD and MULT relate numeric terms of different kinds.

  AND related expressions of the same kind.

(34) 1+2, 10X20

(35) a. dix-huit, deux-cent

‘eighteen’ ‘two hundred’

b. *dix-vingt, *cent-trois-cent”

‘ten twenty’. ‘one humdred tree hundred’

c. John and Mary came in.

d. John and all the students came in.

e. *John and I love Mary came in.

Some differences in the interpretation of ADD/AND

- ADD conjunction is interpreted as the sum of its parts; this is not necessarily the case for AND.

(36) vingt-et-un , 20 + 1 = 21, and not the set {20, 1}

(37) Paul et Marie, the set {Paul, Marie}

(38) one plus two, one and two

(39) a. Barbara and Mat went to the market.

b. Barbara went to the market and Mat went to the market.

c. Barbara and Mats wrote an article together.

d. wrote-an-article-together(Barbara @ Mats)

Numeral ADD conjunctions are distinct from phrasal AND conjunctions, notwithstanding the fact that their functional heads can be pronounced by *and/e/et*.

Non Boolean and

While Boolean and non Boolean conjunction are observed in syntactic expressions, the coordinating conjunction in complex numerals (ADD) is a non-Boolean conjunction, in the sense of Krifka (1983). The conjunction of two numerals is interpreted as the sum of their parts.

To capture its semantics, Masscy (1976), Link (1983), and Hoeksenl(1983). propose an operation which maps entities onto a new entity, their 'sum' or 'collection'.

”$\oplus$” is the joint operation

”$\oplus$" is idempotent ($a \oplus a = a$), symmetric ($a \oplus b = b \oplus a$), and associative ($a \oplus [b \oplus c] = [a \oplus b] \oplus c$)

Some Differences between +/-, ADD/ MULT, ADD/AND

- Addition and multiplication are symmetrical, (40), ADD and MULT are asymmetrical, (31). There are symmetrical and asymmetrical AND conjunctions.

(40) 1+2, 2+1
10X20, 20X10
(41) two hundred, hundred (and) two
deux cent, cent deux (Fr)
'two hundred', 'one hundred (and) two'

- Phrasal conjunctions with an unpronounced head cannot be interpreted as the product of their parts; whereas this is the case for complex numerals with an unpronounced MULT head.

(*42*) a. les nombres deux, cent et mille (Fr.)
'the numbers two, one hundred, and one thousand'
(2, 100, 1,000)
b. deux cent mille (Fr.)
'two hundred thousand'
(2 x 100) x 1,000).

Summary so far

- First, while Boolean and non Boolean conjunction are observed in syntactic expressions, the coordinating conjunction in complex numerals (ADD) is a quasi non-Boolean conjunction.
- Second, some phrasal AND conjunctions are symmetrical, numeral ADD conjunctions are asymmetrical only.
- Third, phrasal AND conjunctions cannot be interpreted as the sum of their parts, numeral ADD conjunctions must be.
- Fourth, phrasal conjunctions with an unpronounced head cannot be interpreted as the product of their parts; whereas this is the case for complex numerals with an unpronounced MULT head.

Unpronounced heads

A striking fact about numerals is that while the *addition* operation ADD is in some case pronounced by the coordinating conjunction *and*, the *multiplication* operator MULT never is in the languages under consideration.

(43) a. twenty two
b. two hundred
c. two hundred (and) twenty three
two thousand two hundred (and) twenty three

This contrasts sharply with the interpretation of covert elements in phrasal syntax, which cannot be interpreted as addition or multiplication. The name of these operations must be pronounced in phrasal syntax, whereas this is not the case in numeric expressions, which name the results of these operations.

(44) a. twenty plus two
b. two times one hundred
c. two time one hundred

Complex numerals include unpronounced heads ADD and MULT legible at the conceptual interface between language and mathematics.
This is not the case for unpronounced heads in phrasal syntax.

Brain imaging studies

Friederici, Bahlmann, Friedrich & Makuuchi's (2011) brain imaging results indicte that Processing hierarchically structured mathematical formulae and processing complex syntactic hierarchies in language activates different areas of the brain.

Language is a faculty specific to humans. It is characterized by hierarchical, recursive structures. The processing of hierarchically complex sentences is known to recruit Broca's area.

Comparisons across brain imaging studies investigating similar hierarchical structures in different domains revealed that complex hierarchical structures that mimic those of natural languages mainly activate Broca's area, that is, left Brodmann area (BA) 44/45, whereas hierarchically structured mathematical formulae, moreover, strongly recruit more anteriorly located region BA 47.

The present results call for a model of the prefrontal cortex assuming two systems of processing complex hierarchy: one system determined by cognitive control for which the posterior-to-anterior gradient applies active in the case of processing hierarchically structured mathematical formulae, and one system which is confined to the posterior parts of the prefrontal cortex processing complex syntactic hierarchies in language efficiently.

Two pathways

MERGE from a cortical network / working memory perspective.

There would be a dichotomy to reason either geometrically or equivalently algebraically in mathematics.

This would imply that MERGE is subserved crucially by two pathways, the one for syntax would pass via the ventrolateral prefrontal cortex, strongly relying on Broca's area, and the other implementation for general reasoning which could integrate multi-sensory information via a dorsal fronto-parietal network, with a strong involvement of the posteriorparietal cortex and the angular gyrus.

Processing complex numerals

We predict that the processing complex numerals mainly activate Broca's area.
The fact that the derivation of numerals shares properties with the derivation of syntactic objects, hierarchical structures, while they interface with different cognitive subsystems than syntactic objects would bring support to the hypothesis that the processing of arithmetic operators is biologically based in the language faculty. Their interpretation however would be determined by cognitive control.

There is evidence that complex numerals and syntactic expressions are derived by Merge and the recursive procedure of the language faculty, while their interpretation activates the neuronal network differently.

The fact that the derivation of numerals shares properties with the derivation of syntactic objects while they interface with different cognitive subsystems than syntactic objects brings support to the hypothesis that arithmetic operations are biologically grounded in the language faculty, while their interpretation access different sub-systems of cognition.

Summary

We raised the following questions:

What is the computational procedure that derives complex numerals?

How are complex numerals interpreted by the external systems?

We argued that the procedure that derives numerals is Asymmetric Merge and indirect recursion via unpronounced operators.

The hypothesis that mathematics emerged with Merge offers a rationale to the fact that Merge and recursion are observed in arithmetic as well as in language.

However, it might be the case that numerals and phrases are interpreted by different sub-system of the cognition.

The fact that MULT is never pronounced indicates that these operators are accessed directly by the part of the cognitive system that sub-serves mathematics.

Then, numerals would find their biological basis in the neuronal faculty that sub-serves grammar but goes beyond it.

Concluding remarks

Our hypothesis fits in well with comparative studies of mathematical capabilities in nonhuman animals: many animals can handle with numbers up to 6-7,, but they cannot deal with greater numbers, and they do not have the language faculty.

This kind of exploration is possible in a framework that takes the language faculty to be a generative procedure deriving the discrete infinity of language. It supports the view that complex numerals emerged with Merge and the generative procedure of language faculty.

It offers a rationale to the fact that Merge and recursion is observed in arithmetic as well as in language, while arithmetic and syntactic expressions are interpreted differently.

Complex numerals find their biological basis in the neuronal faculty that sub-serves grammar but goes beyond it.

References

Beldade, P., Koops, K. & P. Brakefield 2002. Modularity, Individuality, and evo-devo in butterfly wings. *PNAS* vol. 22: 14262-14267

Bickerton, D. 1990. *Language and Species*. Chicago: University of Chicago Press.

Bickerton, D. 1998. Catastrophic evolution: The case for a single step from protolanguageto full human language. In J.R. Hurford, M. Studdert Kennedy & C. Knight (eds.),*Approaches to the Evolution of Language: Social and Cognitive Bases*, 341-358.Cambridge: Cambridge University Press.

Bolker, J. 2000. Modularity in animal development and evolution: Elements of aconceptual framework for EvoDevo. *American Zoology* 40:770-776.

Chomsky, N. (2011). Poverty of the stimulus unfinished business. To appear in A.M. Di Sciullo (ed.) *Interfaces in Language Design*.

Chomsky, N. (2005). Three factors in language design. *Linguistic Inquiry* 36:1-22.

Chomsky, N. 2001. Derivation by phase. In Michael Kenstowicz (ed.), *Ken Hale: A Life in Language,* 1–52. Cambridge, MA: MIT Press.

Chomsky, N. 1995. *The Minimalist Program*. Cambridge, MA: MIT Press.

Chomsky, N. 1988. *Language and Problems of Knowledge*. Cambridge, Mass. and London : The MIT Press.

Chomsky, N. & R. Berwick. 2011. The Biolinguistic program. The current state of its idevelopment. In A.M. Di Sciullo & C. Boeckx (eds.). *The Biolinguistic Entreprise: New Perspectives on the Evolution and Nature of the Human Language Faculty*. Pp13-45. Oxford: Oxford University Press.

Comrie, Bernard. 2005. Numeral bases. In M. Haspelmath, M. Dryer, D. Gil and B.Comrie (eds.), The world atlas of language structures , Chapter 131. Oxford: Oxford University Press.

Coolidge, F. L. and T. Wynn, 2005. Working memory, its executive functions, and the emergence of modern thinking. *Cambridge Archaeological Journal* 15(1):5-26.

Corbett, G. G. (1978). Universals in the syntax of cardinal numerals, *Lingua*, 46:355-368.

Corver, N. 1998. Predicate Movement in Pseudopartitive Constructions. In A. Alexiadou& C. Wilder (eds.) *Possessors, Predicates and Movement in the Determiner Phrase*, John Benjamins.

Di Sciullo, A.M. 2011. A biolinguistic approach to morphological variation. In A.M. DiSciullo & C. Boeckx (eds.). *The Biolinguistic Entreprise: New Perspectives on the Evolution and Nature of the Human Language Faculty*. Pp. 305-326. Oxford: Oxford University Press.

Di Sciullo, A,M. et al. The biological nature of human language. Biolinguistics.

Friedrich, R. & A.D. Friederici. (2009). Mathematical Logic in the Human Brain: Syntax. *PLoS ONE* 4(5): e5599.

Gelman, R. and S. Cordes. 2001, Counting in animals and humans. In E. Dupoux (Ed.).(2001). *Essay in Honor of Jacques Mehler.* Cambridge, MA: MIT Press.

Hauser, M., Chomsky, N., & T. Fitch. (2002). The faculty of language: what is it, who as it, and how did it evolve. *Science* 198. 1569-79

Hauser, M.D 2000. What do animals think about numbers? AMERICAN SCIENTIST 88 (2): 144-151.

Hauser. M, D,, MacNeilage, P., Ware, M. 1996. Numerical representations in primates. PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES OF THE UNITED STATES OF AMERICA 93 (4): 1514-1517.

References

Ionin, T. & O. Matushansky. (2006). The composition of complex cardinals. Journal of Semantics 23: 315–360.

Hurford, J. R. (2003). The Interaction between Numerals and Nouns. In: F. Plank (ed.) *Noun Phrase Structure in the Languages of Europe, Empirical Approaches to Language Typology*. Mouton de Gruyter, pp. 561–620.

Hurford, J. R. (1998). The interaction between numerals and nouns

Hurford, J. R. (1987). *Language and Number: The Emergence of a CognitiveSystem*. Oxford: Basil Blackwell.

Hurford, J. R. (1975). *The linguistic theory of numerals*. Cambridge: Cambridge University Press.

Li, C. (1975). (ed.) *Word order and word order change*. Austin, Texas: University of Texas Press.

Mengden, F. von. (2010). Cardinal Numerals. Old English from a Cross-Linguistic Perspective. Berlin, New York (De Gruyter Mouton) 2010

Munn, Alan. 1987. Coordinate structure and X-bar theory. McGill working papers in linguistics 4:121140. Munn, Alan. 1993. Topics in the syntax and semantics of coordinate struc- tures. Doctoral Dissertation.

Stump, Gregory. 2010. The derivation of compound ordinal numerals: Implications for morphological theory. Word Structure 3.2: 205-233

Greenberg, J. H. (1975). Dynamic aspects of word order in the numeral classifier, in Li, 27-45.

GvozdanoviIc, J. (ed.) (1992). *Indo-European numerals*, (Trends in Linguistics,Studies and Monographs 57, ed. Werner Winter). Berlin: Moutonde Gruyter.Hauser, M., Chomsky, N. and T. Fitch (2002)

Hurford, J. R. 2003. The Interaction between Numerals and Nouns. In F. Plank (ed.),

Noun Phrase Structure in the Languages of Europe 561-620. Berlin:Walter de Gruyter.

Hurford, J. R. (1998). The interaction between numerals and nouns

Hurford, J. R. (1987). *Language and Number: The Emergence of a CognitiveSystem*. Oxford: Basil Blackwell.

Hurford, J. R. (1975). *The linguistic theory of numerals*. Cambridge: Cambridge University Press.

Li, C. (1975). (ed.) *Word order and word order change*. Austin, Texas: University of Texas Press.

Mengden, F. von. (2010). Cardinal Numerals. Old English from a Cross-Linguistic Perspective. Berlin, New York (De Gruyter Mouton) 2010

Stump, Gregory. 2010. The derivation of compound ordinal numerals: Implications for morphological theory. Word Structure 3.2: 205-233